香港國際數學競賽初賽 2019（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD
HEAT ROUND 2019 (HONG KONG REGION)

中學一年級 Secondary 1

時限：60 分鐘

Time allowed: 60 minutes

試題
Question Paper

考生須知:

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. 本卷共 5 個範疇，每範疇有 4 題，共 20 題，每題 3 分，總分 60 分，答錯不扣分。
   There are 5 exam areas and 4 questions in each exam area. There are a total of 20 questions in this Question-Answer Book. Each carries 3 marks. Total score is 60 marks. No points are deducted for incorrect answers.

3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.

4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.

5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.

6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

本試題不可取走。
THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。
DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

填空題 (第 1 至 20 題) (每題 3 分，答錯及空題不扣分)

Open-Ended Questions (1st ~20th) (3 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given *A* and *B* are two non-zero digits and the 2-digit numbers formed by these two digits have the following properties:

   1. $\overline{BA}$ is a prime number greater than 30;

   2. $\overline{AB}$ only has 1 prime factor;

   Find the 2-digit number $\overline{BA}$.

   已知 *A* 和 *B* 為兩個非零數位，且利用這兩個數位組成的兩位數有以下性質：

   1. $\overline{BA}$ 是大於 30 的質數；

   2. $\overline{AB}$ 只有一個質因數，而不是質數；

   求兩位數 $\overline{BA}$ 。

2. Alice goes west for 60km, then goes north for 25km. How far is she now from the original position?

   愛麗絲向西走了 60 公里，向北走了 25 公里，問她和原來位置相距多遠？

3. How many time(s) does hour hand and minute hand of a clock form a right angle for a day?

   每天時鐘的時針和分針會形成直角多少次？

4. Alice is saving money. She save \$1 on the first day, \$3 on the second day, \$5 on the third day, etc, until the total money she saved are multiples of 60. How long does she need to save?

   愛麗絲在儲錢，第一天儲 1 元，第二天儲 3 元，第三天儲 5 元，如此類推。直至儲了的總金額為 60 的倍數，問她要儲多少天？

Algebra
代數

5. Find the value of *x* if $(81+10x)-(6x-15)=0$.

   若 $(81+10x)-(6x-15)=0$，求 *x* 的值。

6. If *x* and *y* are positive integers and $3x-9y=27$, find the minimum value of $x-y$.

   若 *x* 和 *y* 是正整數且 $3x-9y=27$，求 $x-y$ 的最小值。

7. How many integral solution(s) is / are there for *z* if $662>7z+18>-31$?

   若 $662>7z+18>-31$，則 *z* 有多少個整數解？

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

8. Given a and b are positive real numbers and satisfy equations $a^2 - b^2 = 128$ and $a = 8 - b$, find the value of $b - a$.

   已知 a、b 皆為正實數且滿足等式 $a^2 - b^2 = 128$ 和 $a = 8 - b$，求 $b - a$ 的值。

Number Theory
數論

9. Find the sum of all positive factor(s) of 2023.

   求 2023 的所有正因數之和。

10. From 1 to 2019, how many integer(s) is / are divisible by 3 and 7, but not divisible by 6?

    由 1 至 2019 中，有多少個整數可同時被 3 和 7 整除，但不可被 6 整除?

11. Find the unit value of $2^{200} + 3^{300} + 4^{400}$.

    求 $2^{200} + 3^{300} + 4^{400}$ 的個位值。

12. Given $x > 0$ and $x + \dfrac{1}{x} = 4$, find the value of $x^3 + \dfrac{1}{x^3}$.

    已知 $x > 0$ 及 $x + \dfrac{1}{x} = 4$，求 $x^3 + \dfrac{1}{x^3}$ 的值。

Geometry
幾何

13. A cube with side length 8 is cut into cubes with side length 2. Find the increase in total surface area of the cubes.

    一個邊長8的正方體被切成若干個邊長為2的正方體，總表面面積一共增加了多少?

14. An iron wire is bent into two circles whose radiui are 14. If the wire is now bent into 4 identical rectangles, what is the maximum value of area of each rectangle? (Take $\pi = \dfrac{22}{7}$)

    一條鐵線被屈成兩個半徑為 14 的圓形，若這條鐵線現在被屈成四個相同的矩形，每個矩形的面積最大是多少?（取 $\pi = \dfrac{22}{7}$)

15. Small cubes with side length 1 are combined according to the pattern shown below. If there are 28 layers, find the volume.

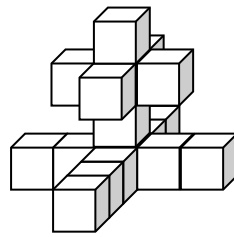   邊長為 1 的小立方體按以下規律組合成立體，如果它有 28 層，求該立體的體積。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Question 15
第 15 題

16. The hypotenuse of a triangle is 10, find the maximum area of the triangle.
一個三角形的斜邊長為 10，求這直角三角形面積的最大值。

Combinatorics
組合數學

17. For a n-digit number which leftmost digit is 3, if the leftmost digit is now put at the rightmost, the new number formed is 3 times as the original one. Find the minimum value of n.
有一個 n 位數的最左邊數位是 3，若把最左邊的數位放在最右邊，則組成的數字會是原來的 3 倍。求 n 的最小值。

18. 1st April, 2019 is Monday, which year will 1st April be Friday for the earliest case?
2019 年 4 月 1 日是星期一，之後最快哪一年的 4 月 1 日會是星期五？

19. In how many possible ways can 10 identical flowers be distributed to 3 distinct vases with at least one in each vase?
把 10 支完全相同的花放入三個不同的花瓶中，每個花瓶至少有 1 支花，問共有多少個不同的分配方法？

20. Use 2, 4, 5, 6, 8, 9 to form six-digit numbers. The odd digits cannot be next to each other. How many number(s) can be formed in this way?
使用 2、4、5、6、8、9 中組成六位數，令奇數位值互不相鄰。問這樣的數有多少個？

~ 全卷完 ~
~ End of Paper ~